5^{th} Polish-German workshop on dynamical problems in mechanical systems
August 31^{st} - September 7^{th} in Zakopane, Poland

Numerical analysis of long-term wear models

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Dedicated to Professor Dr. R. Bogacz on the occasion of his 60^{th} birthday

Abstract

We study options for the computer simulation of wear processes in rail-wheel contact. The feedback loop between dynamics of the rolling body or multi-body system and long-term evolution of the contact geometry causes considerable numerical difficulties. Thus we propose a phenomenological approach as an alternative to a direct solution of the coupled model equations. A relation between curvature of the contact surface and the speed of abrasion allows to establish a nonlinear evolution equation for the wheel radius. After regularisation we obtain numerical solutions which are qualitatively consistent with observed long-term behaviour of real railway wheels.

Introduction

Railway wheels carry considerable load - of the order of 10^5 N - on a small contact patch with the rail. This results in stresses which come close to or exceed the yield stress of steel. Additionally to the normal force there is also dry friction between the contact partners. Consequently, wheel surfaces undergo material and geometrical changes with time and assume a typical wear profile. Such a profile is already mimicked by new wheels.

Numerical simulation of the wear process is complicated by the fact that the contact partners are parts of a multi-body system with a non-trivial dynamical behaviour. For realistic speeds there is no stable uniform solution to the equations of motion, cf. Kaas-Petersen 86. Instead we observe pseudo-periodic trajectories. This lateral hunting of a wheelset is limited by the flanges of the wheels. The presence of flanges limits the applicability of linear models and leads to additional complications in the contact geometry, cf. Netter 93, Frischmuth et. al. 94. However, in recent years suitable methods for an efficient numerical treatment of this type of problems have been developed, cf. Arnold and Frischmuth 97, Frischmuth 96.

The long-term behaviour is much less understood. Certain physical quantities like hardness, intensity of microcracks and surface roughness are certainly involved in the wear process. But till now there is no ultimate explanation of the mechanism which leads in the end to the removal of tiny chops of material out of the contact surface and thus to a slow reduction of the wheel's radius. Most amiable for numerical simulation is the assumption that the removed mass is proportional to the frictional energy dissipated in the contact process, cf. Brommundt 96 and Frischmuth 97. But we should be aware that this is only a rough simplification which ignores the internal changes of the material. Our model should be open for more detailed wear laws as well as for more convenient ones.

Given both components of a coupled model - equations of motion and wear law - performing numerical calculations is still quite demanding. While the dynamics of a wheelset is characterised by a time scale of about one second, the wear process takes several weeks or even month before changes can be detected. A straightforward integration of the coupled equations over appropriate time intervals would be not only very time consuming but also little reliable with regard to numerical errors.

A typical way to close the gap between the time scales is to increase the coefficient in the wear law and to reduce accordingly the length of the time interval. Because this approach depends on the choice of the acceleration factor while it is still computationally expensive we prefer to look for a way to avoid dynamical simulations during the long-term integration.

For linear dynamical systems this can be done by Fourier techniques, cf. Brommundt 96. In the general case we try to find relations between the shape of the wear surface and the intensity of relevant factors which determine the further evolution. To this end we have to carry out numerical calculations with the dynamical model for frozen geometry, but we do this off line.

This fact is most essential for the very same reason which attracts so much attention to the considered wear problems: during the process the wheel looses its axial symmetry. Typically, some 3 to 8 minima and maxima of the radius occur in angular direction. This, of course, adds to the numerical cost of the dynamical sub-model. It should be pointed out that the equations of motion are very sensitive to geometrical changes. If modeled as rigid multi-body system, the differential-algebraic equations of motion have index 3, which implies that second derivatives of perturbations in the constraints (defined by the geometrical changes) influence the perturbations in the solution, cf. Arnold 97, Arnold and Frischmuth. 97, Hairer 91, Simeon et. al. 91. Also for elastic contact models small geometrical modifications can result in considerable effects, e.g. on frictional forces and dissipated energy.

In this paper, we postulate a nonlinear wear law relating the speed of abrasion to curvature and speed of the guiding motion. Numerical studies of the resulting partial differential equation exhibit the behaviour typical for observed wear processes. We obtain a stable evolution of an axial symmetric surface for small speeds, while for higher speeds initial deviations from symmetry lead to "polygonalization". This effect is achieved without assuming inhomogeneous material, and it is not just an amplification of a pre-existing unroundness.

Dynamics of the short time scale

Within this paper we restrict ourselves to a differential algebraic model of the short time dynamics of the system under consideration. Each body involved is assigned 6 degrees of freedom which are later on restricted by constraints, e.g. by the contact geometry or by the guiding motion. Bodies can be connected by elements like springs, dampers and joints. For details we refer to Netter 93 or Simeon et. al. 91.

We collect all unknown position co-ordinates into a vector y. Equations of motion and constraints are expressed by the Euler-Lagrange system

$$\begin{aligned} m(y)\ddot{y} &= f(t, y, \dot{y}, \lambda) + \nabla g^T \lambda, \\ 0 &= g(t, y). \end{aligned} \quad (1)$$

In Figure 1 a typical solution for the motion of a wheelset on a straight track is presented. Above a certain critical speed the trivial solution to (1) becomes unstable, and solutions tend to a limit cycle.

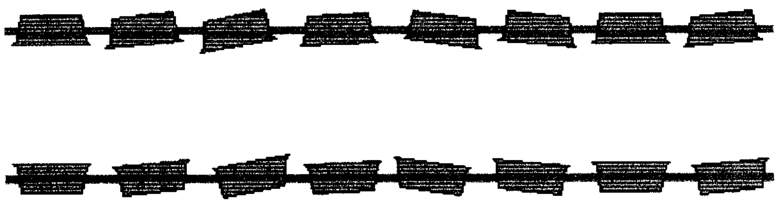


Figure 1: Hunting of a wheelset

We omit a detailed discussion of the system (1), however it must be mentioned that the dependence of f on the Lagrange multiplier λ expresses a friction law. There are various approaches to determine the dependence of tangential forces on normal forces, positions y and velocities $\dot{y}$. In this paper we based all calculations on Kalker's linear theory, cf. Kalker 90.

The law of wear

From a solution to (1) we can obtain among other data the position of the geometrical contact point x_c, the normal force λ and the power π of the frictional forces as functions of time t. We adopt here an internal state variable approach restricted to the wear surface (respectively circumference for 2D disk wheels). As parameters describing the actual state of a boundary point we introduce a pair $(u,z)^T$, where u is the actual radius reduction and z comprises all remaining relevant state parameters.

The kinetic equations describing the temporal evolution of the state are given by

$$\begin{aligned} \dot{u} &= U(t,\pi(t),u,z), \\ \dot{z} &= Z(t,\pi(t),u,z). \end{aligned} \qquad (2)$$

Here we denoted by π quantities responsible for the external forcing of the wear process, e. g. the frictional power, normal or tangential force.

In the simplest case the collection of hidden variables z may be empty, i.e., U is a function of the remaining variables alone. As a special case we obtain the classical abrasion law with

$$\dot{u} = U(\pi(t)) = -\beta\pi(t). \qquad \text{(2a)}$$

The parameter β is a positive constant and π the power of the friction forces.

Coupling effects

The coupled model equations consist of the equations of motion (1) for the short time dynamics together with the abrasion model (2) or for simplicity (2a).

$$\begin{aligned} m(y)\ddot{y} &= f(t,y,\dot{y},\lambda) + \nabla g^T\lambda, \\ 0 &= g(t,y,u), \\ \dot{u} &= -\beta\pi(t,y,\dot{y},\lambda)\chi(\cdot\ ;t,y,\dot{y},\lambda) \end{aligned} \qquad (3)$$

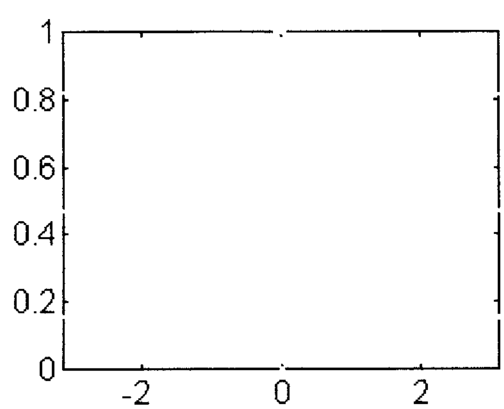
But now the constraint function g in (1) depends on the variable u while the right-hand side of the kinetic equation for u depends on the quantities π and χ which are determined from the variables of the dynamical part of the system, in particular from λ. For convenience we formulate only the system (3) based on the classical abrasion hypothesis.

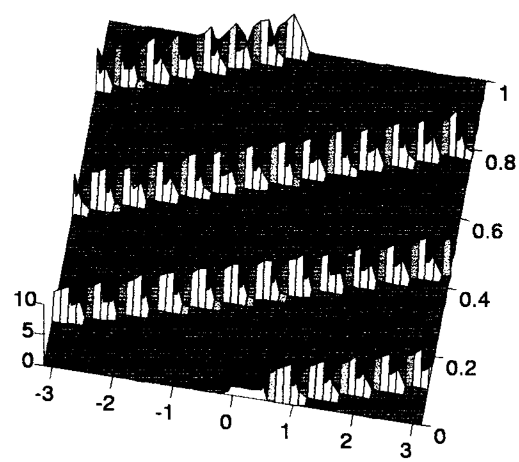
It is essential to notice that u is a function of time t and surface position, i.e. of two or more variables. The function χ determines the spatial distribution of the wear due to frictional power over the contact patch on the wear surface.

However, due to computational restrictions, wear models discretize usually not fine enough to resolve the contact patch. Hence the wear is just associated with the geometrical point of contact.

Figures 2 and 3 show a short fragment of a typical solution. The line in Figure 2 represents the trajectory of the geometrical contact point. In Figure 3 the angular distribution of wear is shown.

Typically, wear is non-negative and has a very small support. Oscillations along the trajectory are due to short time dynamics, e.g. hunting or elastic oscillations.





Figures 2 and 3: Contact point and frictional power over angle and time.

For the discretization shown in Figure 4 the size of the contact patch is much smaller than the panels in the wear surface. A reasonable resolution of the contact zone requires several hundred grid cells in angular direction, so that such calculations are mostly restricted to simplified models like the disk wheel in Figure 5.

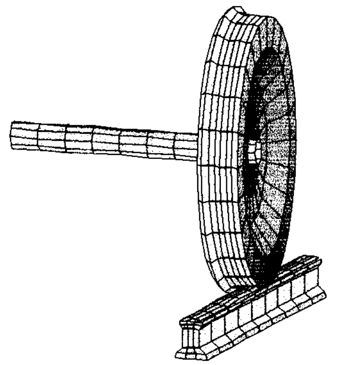


Figure 4: Discretization of the wear surface.

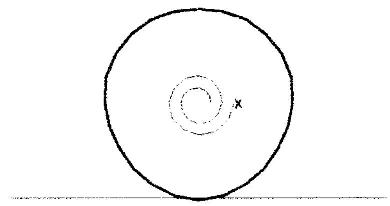


Figure 5: Simplified model of a disk wheel.

Solution strategies for the coupled equations

Coupled problems with components differing in their characteristic time scales are not uncommon. For instance, in hydrology the nonlinear interactions between surface waves and the bottom topography of coastal water bodies are investigated, cf. Restrepo 95. Often there are also effects of different spatial scales involved, e.g. we can have interactions between molecular dynamics and macroscopic motion. For an example we refer to Frischmuth and Hänler 97. Even for a moderate relation between two time scales of a problem numerical oscillations may result as shown for the second sound model in Frischmuth and Cimmelli 97. Such numerical effects can easily be mistaken for a pattern of the modeled phenomenon. Note that for the polygonalization radial deviations are less than one per cent of the wheel diameter, which is for wear profiles about the accuracy of the contact geometry, cf. Netter 93.

If there is an analytical solution to the dynamical sub-problem, e.g. if (1) is a linear system, then the coupled problem simplifies drastically. In Brommundt 96 a Fourier technique was developed for this class of models. We confirmed the results of Brommundt 96 by a similar spline approach. The resulting radius distribution over angle and time is shown in Figure 6.

Note that initial maxima of the radius may be levelled with usage as well as new minima can develop rapidly.

Throughout the remaining part of this paper the boundary co-ordinate x is always an angle running through an interval of length 2Π while the time varies over the normed life-time interval $[0,1]$.

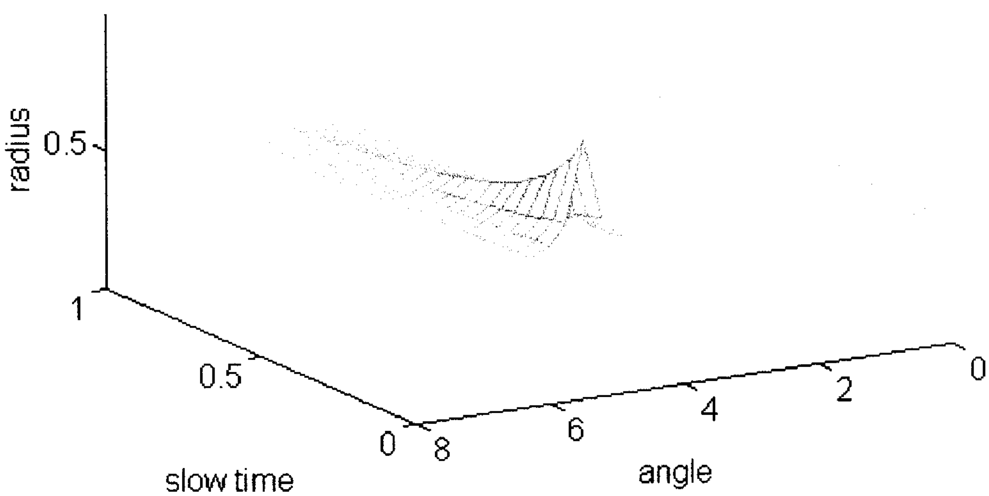


Figure 6: Wear for linear dynamics.

In the general case however, a look at the wear intensity at a fixed boundary position suggests another approach.

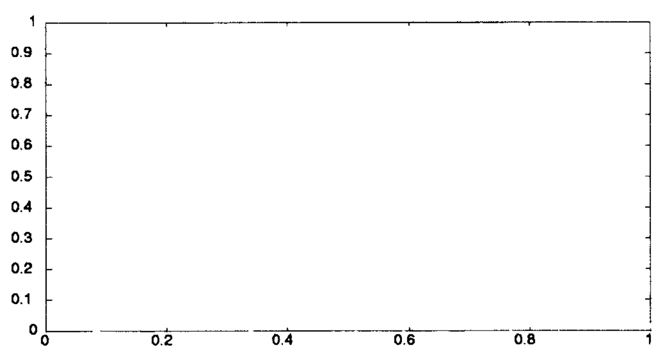


Figure 7: Wear intensity versus time at fixed position.

Figure 7 displays just 12 turns of a disk wheel. It is obvious that the time integration over the wear intensity - which gives the actual decay of the radius - should not be done by a method for smooth integrands. Instead it seems very sensible to take advantage of a Monte Carlo type method. All we really need is the distribution of the integrals of the "teeth" of the above curve. This leads to a stepwise procedure for solving the coupled system. For a suitable set of nodal points on the surface curves like in Figure 7 are calculated with frozen parameters. After appropriate integration time the result is extrapolated to a relevant time step in terms of the long term process.

Numerical tests have been carried out with this method for various initial geometries. For details and results we have to refer to the forthcoming paper Langemann 97.

In the next section, we try to establish a relation between mean wear intensity and local wheel geometry at the surface point under consideration.

A phenomenological model

The idea of this section is to relate mean values of the influence factors π to some local quantities instead of calculating them as time functions and integrating later. In general, this proves to be as difficult as to derive macroscopic constitutive equations from microscopic considerations. However, for some special cases and suitable assumptions a relation can be deduced rather than postulated. From those cases we generalise in an inductive step and try to fit the resulting model to observed behaviour of the real system.

For a rigid disk wheel we can solve the equations of motion explicitly. If we neglect higher order terms in the slope of the radius, the Lagrange multiplier λ is - up to the constant load - the product of squared speed and curvature u_{xx}. The mean slip - appearing in friction law as well as in the frictional power - depends on the normal force, i.e. on λ. After randomisation, higher normal force corresponds to smaller slip. Note that there is in general no pointwise dependence of that type.

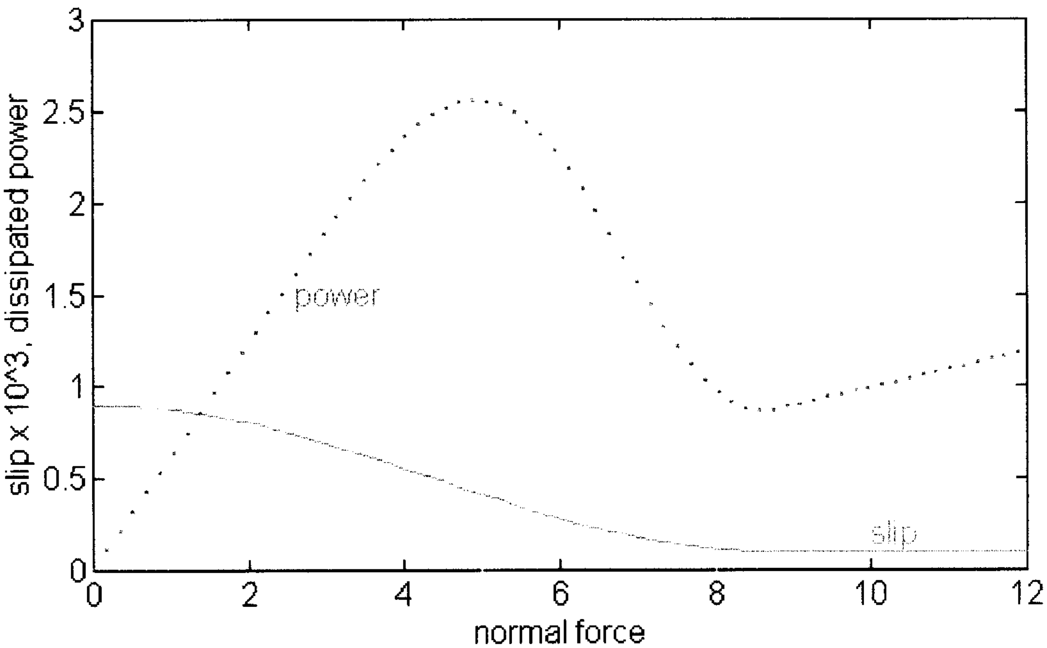


Figure 8: Relation between mean values of influence factors.

For the details of the randomisation, especially in the case of realistic wheels, we have again to refer to Langemann 97. We remark only that the estimation of realistic distributions of rotational and translational velocities - which have typically various peeks - is most essential. After curve-fitting we arrive at an evolution equation of the form

$$\dot{u} = h(u_{xx}; u, u_x, v) \qquad (4)$$

where v is a parameter describing the guiding motion, e.g. the vehicle speed.

Discussion of the evolution equation

The main feature of the above equation is that the right-hand side is non-monotonous in its first argument. On increasing branches of h solutions to (4) behave like temperature fields. Hence we cannot expect instabilities. On the other hand, a decreasing dependence on u_{xx} indicates ill-posedness. Hence a regularization is necessary. For an analytical study of (4) together with detailed numerical experiments we refer to Hänler 97. For this paper we restrict ourselves to the presentation of several selected results. Figures 9 and 10 compare initial and final deviations u from the nominal radius as functions of the angle x.

For small speeds v the uniform geometry of a disk wheel remains stable, only the mean radius decreases in time. A slightly higher speed produces a smoothing of small initial perturbations.

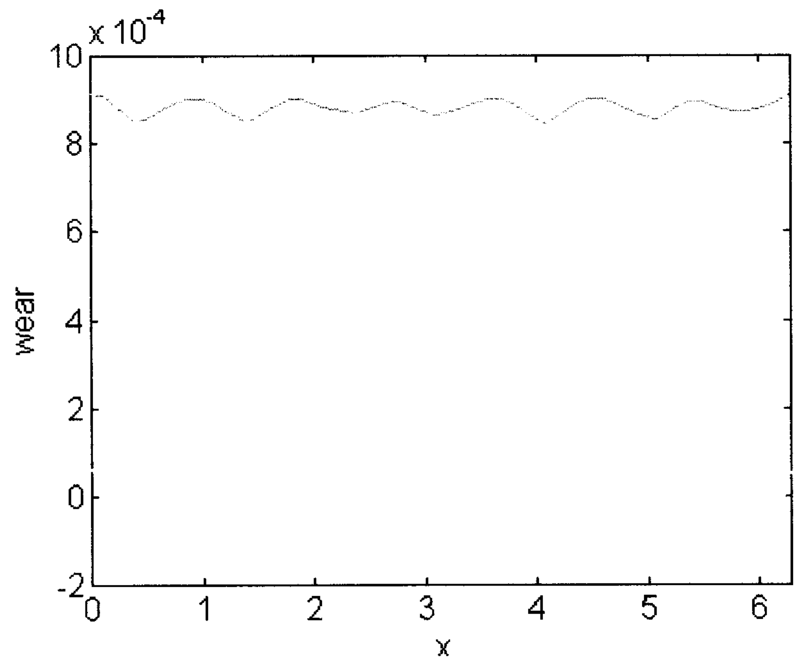


Figure 9: Stable behaviour at small speeds.

Increasing speed leads to the onset of instability.

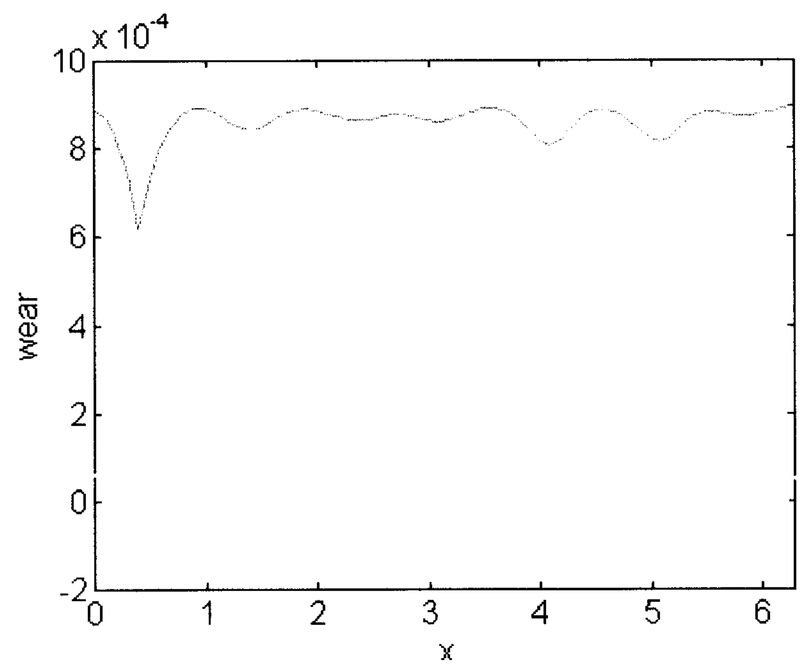


Figure 10: Onset of instability.

At still higher speed, stability is lost. Some local minima of u remain almost untouched. The radius stays close to its initial value. However, it depends on the global state and parameters which places wear off and which not. Figure 11 shows the evolution until about 1mm of material is removed at the most worn spots.

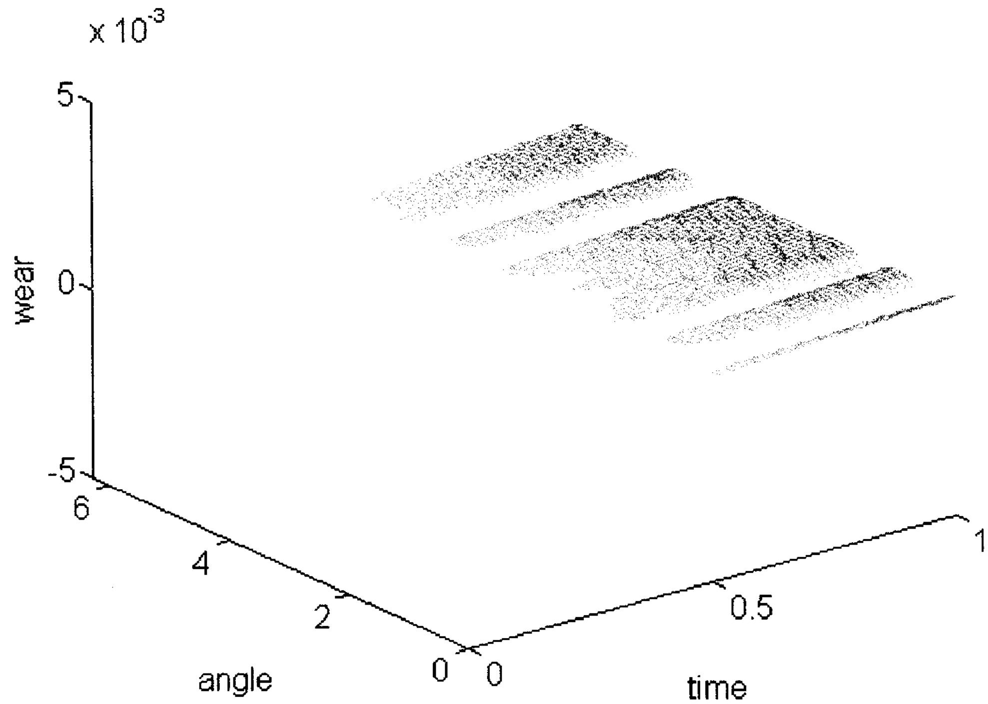


Figure 11: Polygonalization for higher speed.

Conclusion

Direct numerical solution of coupled wear models of railway wheels encounters serious difficulties. Not only computational cost, but most of all accuracy problems render alternative approaches more attractive. A phenomenological evolution model of the geometry of a disk wheel shows qualitatively correct behaviour of the solution. The proposed method can be applied, mutatis mutandis, to more realistic models. Numerical solutions to the PDE (4) can be computed in acceptable time on usual workstations.

Bibliography

Arnold, M., 1997, Numerical problems in the dynamical simulation of wheel-rail systems, Submitted to *Z. Angew. Math. Mech.*

Arnold, M., and Frischmuth, K., 1997, Solving problems with unilateral constraints by DAE methods, Submitted to *Mathematics and Computers in Simulation.*

Brommundt, E., 1996, A simple mechanism for the polygonalization of railway wheels by wear, *Mechanics Research Communications.*

Brommundt, E., 1996, Polygonalization of a single rolling wheel by wear, *Euromech - 2nd European Nonlinear Oscillation Conference*, Prague, September 9-13.

Frischmuth, K., 1996, Regularization methods for non-smooth dynamical problems,

in: R. Bogacz, G. P. Ostermeyer and K. Popp (editors), *Dynamical Problems in Mechanical Systems IV*, Proceedings of the 4th Polish-German Workshop, July 30 - August 4, 1995 in Berlin, IPPT-PAN, Warsaw, 99-108.

Frischmuth, K., 1997, Rail-wheel contact, dynamical simulation and damage, in: W. Kosinski (ed.), *Environmental and Damage Mechanics*, PSP, Warsaw.

Frischmuth, K. and Cimmelli, V. A., 1997, Numerical solution to coupled thermo-elastical problems with hyperbolic heat equation, Submitted to *Arch. Mech.*

Frischmuth, K. and Hänler, M., 1997, On the closed osmometer problem, Submitted *to Z. Angew. Math. Mech.*

Frischmuth, K., Hänler, M., and Ostrzinski, St., 1994, On motion with non-smooth constraints, in:. K. Frischmuth (ed.), *The dynamical simulation of wheel-rail systems*, Proc. of the First

Workshop on "Dynamics of Wheel-Rail-Systems", held at Rostock University, May, 5th and 6th, 1994, Universität Rostock, FB Mathematik, Preprint 94/21, November 1994.

Hänler, M., 1997, Numerical analysis of a nonlinear evolution equation with application in contact mechanics, forthcoming.

Hairer, E. and Wanner, G., 1991, *Solving Ordinary Differential Equations. II. Stiff and Differential-Algebraic Problems*, Springer-Verlag, Berlin Heidelberg New York.

Kaas-Petersen, Ch., 1986, Chaos in a railway bogie, *Acta Mechanica,* **61**, 89-107.

Kalker, J., 1990, *Three-Dimensional Elastic Bodies in Rolling Contact*, Kluwer Academic Publishers, Dordrecht Boston London.

Langemann, D., 1997, Randomization of wear effects, forthcoming.

Netter, H. and Arnold, M., 1993, Geometrie und Dynamik eines Rad-Schiene-Modells in Deskriptorform mit unstetigen Zustandsgrößen, *Technical Report* IB 515-93-02, DLR, D-5000 Köln 90.

Restrepo, J. M. and Bona, J. L., 1995, Discretization of a model for the formation of longshore sand ridges, *J. Comput. Phys.* **122**, No. 1, 129-142.

Simeon, B., Führer, C., and Rentrop, P., 1991, Differential-algebraic equations in vehicle system dynamics, *Surveys on Mathematics for Industry*, **1**, 1-37.

Captions of figures:

1. Hunting of a wheelset.

2. Contact point (over angle and time).

3. Frictional power over angle and time.

4. Discretization of the wear surface.

5. Simplified model of a disk wheel.

6. Wear for linear dynamics.

7. Wear intensity versus time at fixed position.

8. Relation between mean values of influence factors.

9. Stable behaviour at small speeds.

10.Onset of instability.

11.Polygonalization for higher speed.

Figures 2 and 3 can have a common caption, see page 6.

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last change: Rostock, November 15, 1997