Author(s): Kurt Frischmuth and Michael Hänler

Institution: FB Mathematik, Universität Rostock

Title of Talk: Identification of Boundary Data for the Ekman Model

We consider the partial differential equations

$$\Delta u = f \qquad \text{(Laplace equation)} \qquad (1)$$

and

$$\Delta u - h^{-1}(\nabla h, \nabla u) = g \qquad \text{(Ekman equation)} \qquad (2)$$

on a domain Ω with polygonal boundary $\partial\Omega$. We assume a decomposition of $\partial\Omega$ into connected arcs $\partial\Omega_1$, $\partial\Omega_2 \ldots \partial\Omega_{2k}$ to be given. The solution u of either of the problems is subject to the condition

$$u_{|\partial\Omega_{2l}} = \text{const}, \; l = 1 \ldots k. \qquad (3)$$

(For the Ekman model, h is a given non-negative depth profile, vanishing at the boundary parts with even index.)
On the remaining pieces of $\partial\Omega$ there is no boundary condition given.
Instead, we assume that at on finite set $M \subset \Omega$ we are given (measured) values z_x of $\nabla u(x)$.
We consider the functional

$$\phi(u) := \sum_{x \in M} \|z_x - \nabla u(x)\|^2 + \mu \|u_{|\partial\Omega}\|^2 . \qquad (4)$$

We show (4) to be well-defined and, moreover, to possess a unique minimizer in an appropriate subspace of H
($H = H^1(\Omega)$ for (1) and $H = H_h^1(\Omega) := \{u \in H^1(\Omega) \,|\, h^{-1}\nabla u \in L^2(\Omega)\}$ for (2)).
We apply the Ekman equation (2) and its special case with $h \equiv const$ (1) to an estuary flow problem. In that case the gradients are (rotated by an right angle) velocity vectors, and M contains the positions of beacons.
Numerical solutions to the above identification problems are discussed in dependence on

1. the choice of the set M of measurement points,
2. the regularization parameter μ and
3. the data given at M.

For "reasonable sets" of measuring point M we are able to reobtain solutions to (1) as well as (2) from disturbed values of their gradient at M with an error comparable to the simulated error of the measurement. The approximation becomes better (by several orders) outside a vicinity of the boundary parts with even indices.
The results apply to (nonhomogeneous) diffusion and heat conduction problems as well.